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About two types of Nonstationary Stochastic Processes

Miroshnychenko Sergii[a*], Tkachenko Igor[b], Miroshnychenko Yuryi[c]

[a,b,c]*independent researcher, Mariupol, Lviv, Ukraine*

[a]*Email: ig.fed.955@gmail.com*

[c]*Email: jurmir656@gmail.com*

Abstract

Based on satisfactory results of the early developed analytic approach application to analysis of 3-rd order non-stationary stochastic processes (NSPs), the well known mathematical procedure for describing a NSP in more detail was applied to analysis of the 2-nd order ones. Physical bases of the 2-nd order and 3-rd order NSPs were defined as various periods of the NSP development. Based on numerous experimental data the periods were defined as: incubation (preliminary), intensive development, saturation for a 3-rd order NSP and period of intensive development together with the saturation one for a 2-nd order NSP. The constitutive equation for the 2-nd order NSP was derived and analytically solved. Some basic properties of the solutions were considered. Full absence of an equilibrium specific point for a 2-nd order NSP and existence of oscillating trajectories in the phase spaces were shown. The combination of the revealed variants of a 2-nd order NSP development was explained by an action of the mechanism early proposed for the describing the 3-rd order NSPs development. The obtained results were interpreted in terms of unlimited number of 2-nd order NSPs development in countless Universes similar to ourself.

Keywords: Nonstationary Stochastic Processes.

1. Introduction

There are wide variety of stochastic processes (SPs) in Nature [1-3]. Among them the most known groups are nonstationary (NSP) and stationary (SSP) ones [4]. In the most general case practically all observable SPs are NSPs but include a stationary stage as a component for each one. By the time the theoretical description is well developed namely for SSPs or stationary parts of NSPs. Meantime, there are no a general theoretical model to describe a whole NSP or all its stages. One of the possible theoretical approaches was proposed recently to analyze a whole NSP [5…7] and demonstrated good results for the macroplastic deformation of the poli-crystalline FCC metals [7,8]. The approach was also applied to analysis of some basic features of our visible Universe evolution [9] and demonstrate rather satisfactory results.

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* Corresponding author.

There was used in the works [5-9] commonly accepted mathematical description for SPs [4] which is based on specifying the probability distributions for a SP characteristic at a given time moment together with common distributions of the characteristic at other consecutive time moments. As it known [4] such a procedure is referenced to as the first, second and so on probability distribution for a NSP. In mathematical formulation the above means: $\partial P/\partial s_1$, $\partial^2 P/(\partial s_1 \cdot \partial s_2)$ …. As it was noted [3], the above consequence of the derivatives describe a NSP with increased details. The above rigorous mathematical approach was used recently [5…9] to analyze some basic NSPs in nature: macroplastic deformation of FCC poli-crystals [5…8], our University evolution features [8,9]. As the common peculiarities of the NSPs the development the energy barriers overcoming was considered. Physically, the NSPs were considered as consisting of three stages: preliminary or incubation period; the activated or the maximum intensity period and saturation or finishing period corresponding stopping an NSP. Such notions were confirmed preliminary by the numerous experimental data, especially concerned to the plastic deformation and polymorphic phase transformations. Besides, the modeling results finally obtained [5…9] are in a good accordance with available today relevant notions and experimental data. Particularly, using the proposed approach it is possible to predict the following feature of the Universe evolution: possibility of traveling in time; existence of other Universes, one of which is our visible one; possibility of probably quarks and neutrinos formation of respectively Dark Matter and Dark Energy etc. Mathematically the analysis early performed [5…9] was based on the condition: $\partial^3 P/(\partial s_1 \cdot \partial s_2 \cdot \partial s_3) = 0$ in accordance of the above mentioned three stage structure of the NSPs. Proceeding from the obtained results it is reasonable to expect no applicability of relations having been obtained for the considered earlier [5-9] 3-d order NSPs to the 2-nd order ones which are also rather wide spreaded in nature: spinodal decomposition of some condensed phases, order-disorder transitions etc. Meantime, there are no foundations to ignore the applicability of the general mathematic approach [3] to analysis of NSPs. So, it is necessary to show general differences of all the predictions results.

2. **Constitutive equations for the 3-RD AND 2-ND order nsps in two energy level systems**

Lets recalling the basic notions of the early proposed [5…7] analytic approach to describe a NSP. We proceeded from considering a system, consisting of numerous subsystems, each of them may be in two energy states. One of the state we consider as basic one having the relative lowest energy. The other state has an energy exceeding the first one on an quantity ε . In fact, we consider two energy level system with the energy states: 0 and ε. As an addition to the system description we assumed then and to do so further that the system, together with corresponding subsystems, is under an action of both homogeneously distributed a gradient of an energy applied to the system or a tensile stress σ at an absolute temperature T . Besides, we assumed that an each subsystem may spontaneously transfer from one energy state to the other under the unchanged external conditions e.i. σ and T . Evidently, in the most general case, the two variants of the particular transition $0 \to \varepsilon$ are possible: - through an activated energy state; - directly from the lowest to the high energy level. According to the noted above, we should distinct at least two types of NSPs: the NSPs $0 \to \varepsilon$ with energy barrier overcoming - as the 3-rd order NSPs, and, the NSPs $0 \to \varepsilon$ without the intermediate (activated) state – as 2-nd order one. So, from the mathematic point of view, we should write, respectively: $\partial^3 P/(\partial s_1 \cdot \partial s_2 \cdot \partial s_3) = 0$ and $\partial^2 P/(\partial s_1 \cdot \partial s_2) = 0$ where P we denote as the probability to find a subsystem in the high energy state, having energy ε. Here and above we are using $s_{1,2,3,} \equiv s_i$ as a NSP characteristic, values of which change with time t with a finite velocity : $\partial s_i/\partial t =$

$s_i' \neq \infty$. Proceeding from the above, we may write, respectively: $\partial^3P/(\partial s_1 \cdot \partial s_2 \cdot \partial s_3) = (\partial^3P/\partial t^3) \cdot 1/(s_1' \cdot s_2' \cdot s_3') = 0$; $\partial^3P/\partial t^3 = 0$ and $\partial^2P/(\partial s_1 \cdot \partial s_2) = (\partial^2P/\partial t^2) \cdot 1/(s_1' \cdot s_2') = 0$; $\partial^2P/\partial t^2 = 0$. Using further the obtained [5] dependence:

$$P(s) = exp\,[-s\,(\sigma, t)] = exp\,\{-[\,\bar{\sigma}/\sigma(t)]^{\,n(t)}\}$$

where $\bar{\sigma}$ - is a constant, MPa;

n(t)- is a variable associated with a degree of freedom for a subsystem;

one can obtain the constituting equations from the derivative relations, respectively, for a 3-rd order NSP:

$$d^3s/dt^3 - 3\,(d^2s/dt^2) \cdot (ds/dt) + (ds/dt)^3 = 0 \qquad (1)$$

and, for a 2-nd order NSP:

$$d^2s/dt^2 - (ds/dt)^2 = 0 \qquad (2)$$

An analytic solving the equation (2) gives the following general time dependence for the characteristic $s = s(t)$:

$$s = ln\,\{B/[((t \pm t^*)/t_0) \pm 1]\} \qquad (3)$$

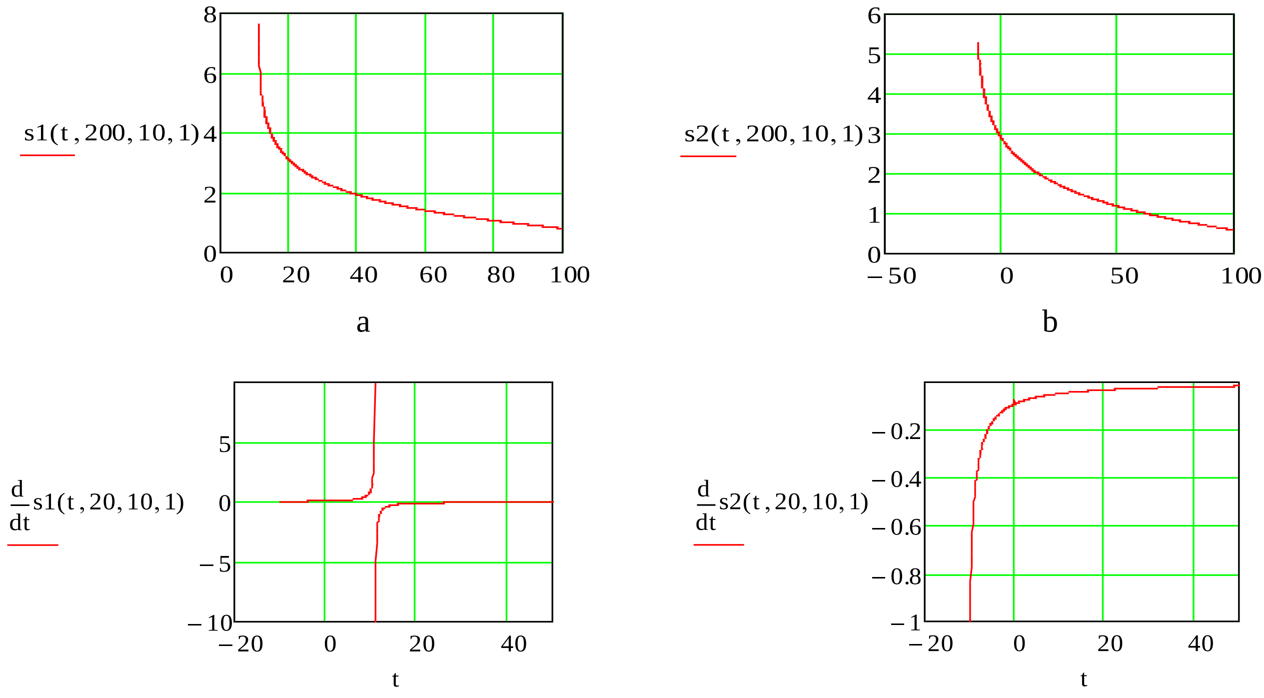
where B, t_0 are integration constants,

t^* is a constant corresponding "shear" of the obtained dependencies along the time axis.

It should be noted that all the constants in relation (3) may be associated with the integration ones, because of a possibility of their mutual transformation. The particular difference between t_0 and t^* is made based on the physical considerations to show some features of a NSP. Based on the above [5-9], from the physical point of view it is possible to associate integration constants with countless numbers of energy carriers providing a NSP development, the number of possible NSPs or systems where the NSPs develop in nature. Graphical dependencies corresponding the equation (3) are shown on Fig.1.

3. Stability analysis of the analytic solutions of the constitutive equation for the 2-ND order nsps

According to the standard procedure of the stability analysis of a differential equation [3], lets consider the differential equation (2) as a system of two following equations:



$s' = ds/dt = P$

$(ds'/dt)=(d^2s/dt^2)= Q$

Figure 1: Calculated time dependencies for the quantities: s1= ln (A/(t - t0) (a) s2= ln (A/(t + t0) (b) and their first time derivatives: ds1/dt (c), ds2/dt (d), at the constants values: A = 200, t0 =10, t* =1 (a,b) ; A = 20, t0 =10, t* =1(c,d)

where *s'*- is a first time derivative of *s*.

As it was ascertained and may be verified using relation (3), the quantities at the right side of the above system, never can reach zero level, excepting values $t \rightarrow \infty$. In other words, the system (2) does not have any special equilibrium point. However, the system may have periodic solutions. Really, according to [4], for the periodic solutions, one can define the quantity λ , which is calculated according to the integral relation, which includes the function:

$dM/ds + dQ/ds'$

As it was noted in [4], if $\lambda < 0$ the periodic trajectories are stable and if $\lambda > 0$ - they are unstable. In other words, it has to be said that if $(dM/ds)+(dQ/ds') < 0$ the periodic solutions correspond stable oscillating trajectories and vise versa. It may be easy to show that the above function value is $-3/(t \pm t_0) < 0$. In other words the function $(dM/ds)+ (dQ/ds')$ or the λ values is negative at any time within the interval $t_0 < t < \infty$ and $-t_0 < -t > -\infty$, but $\lambda > 0$ in the case: $t_0 > t > 0$ and $-t_0 > -t < 0$. So, it may be concluded that periodic 2-nd order NSPs in a corresponding material system have to develop as stable ones or repeat unlimitedly at times exceeding any arbitrary moment t_0 e.g. – at $t > t_0$. Meantime, earlier e.g. at times $0 < t < t_0$ the periodic changes of a system states are unstable.

As may be seen from Fig.1, the 2-nd order NSPs do not have incubation (preliminary) period at any the constant values contrary to the 3-rd order NSP [5, 8,9]

Taking into account inevitable presence of non-periodic time dependence (3) simultaneously with the oscillating trajectories, their combination may be considered as the sporadic, consecutive realization of the such non-perodic solution (3) by analogy with the corresponding mechanism early proposed [6] to the 3-rd order NSP. Periodic realization of the dependencies (3) shows [6] existence of countless systems where the 2-nd order NSPs develop as well as corresponding number of the Universes where the NSPs can develop.

As examples of material systems may having the above behavior, respectively electronic qubits and material alloy phases able to have spinodal decomposition may be considered.

4. Conclusions

1. The commonly accepted mathematical procedure consisting of consecutive partial derivatives calculations for the time evolution of a NSP detailed description was applied to analyze a 3-rd and a 2-nd order NSPs.
2. The constitutive equation for the 2-nd order of NSPs were derived, its analytical solution was obtained and some properties of the solution compared with the corresponding ones for the 3-rd order NSP were analyzed.
3. By comparing properties of the constitutive equation solutions for the 2-nd and 3-rd order NSPs, the absence of the incubation period for the 2-nd order NSP and presence of the period for the 3-rd order one was emphasized.
4. Stability of real solution of the constitutive equation for a 2-nd order NSP was investigated by the standard mathematical procedure and absence of the real specific equilibrium points in the phase space was shown.
5. Existence of oscillating trajectories which are never reach an equilibrium point in a phase space as solutions of the constitutive equation for the 2-nd order NSP was concluded and confirmed by the analyzing stability of the possible periodic solutions.
6. Combination of the non-periodic and oscillating behavior of the analytic solutions of the constitutive equation for the 2-nd order NSP was noted and explained by the mechanism early proposed to act during the 3-rd order NSP.
7. The both stability of the analytic solutions of the constitutive equation for the 2-nd order NSP at times exceeding an arbitrary chosen constant value and their unstable character at lesser times were shown
8. As examples of physical systems corresponding the stable and unstable 2-nd order NSPs within our observable Universe the following material objects was proposed, respectively: qubits; metal alloys of

chemical compositions providing the spinodal decomposition of a matrix phase.

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