

# Temperature Dependence of $\omega$ Meson-nucleon Coupling Constant from the AdS/QCD Soft-wall Model

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## Abstract

We investigate the temperature dependence of the  $\omega$  meson-nucleon coupling constant using the soft-wall model of AdS/QCD at finite temperature. The profile functions for the vector and fermion fields were applied in the model with thermal dilaton field at finite temperature. The interaction Lagrangian was written in zero temperature limit and includes minimal and magnetic type interactions. The temperature dependence graphic  $g_{\omega NN}(T)$  coupling constant is plotted. We observe that the coupling constant become zero near the Hawking temperature.

## 1. Introduction

The study of hadron coupling constants, form factors etc. at finite temperature has a great importance for the investigation of properties of hot hadronic matter . There were several models [1–4] and approaches in order to solve the problems in particle physics and to investigate the processes of hot hadronic matter. One of such models is the soft-wall model of AdS/QCD correspondence [5–8]. Reference [9–19], the phase transition [20], and for the theoretical studies in the hot hadronic matter as well [21–28]. In this paper we shall be interested a knowledge about the temperature dependence of the coupling constants and form factors of the strong interactions between the mesons and baryons in AdS/QCD soft wall model. In generally, the quantum field theory in a confined phase is holographic dual to the gravity in the AdS space - time. In [23, 25, 26] it was considered to the thermal soft-wall AdS/QCD model. Solutions of the equations of motion (profile functions) for the fermion fields interacted with the thermal dilaton field at finite temperature were found in the Reff [25, 26]. It is interesting for us, to know the temperature dependence of meson - nucleon coupling constants of  $\omega$  meson at finite temperature. Getting profile functions for the vector  $\omega$  mesons and using holographic principle this issue can be solved in the framework of the thermal dilaton soft-wall model at finite temperature and we can investigate the temperature behavior of  $g_{\omega NN}(T)$  , when temperature approaches to the phase transition temperature. The remainder of this paper is organized as follows: In Section II is about the soft-wall model at finite temperature.

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Sections III and V we get bulk-to-boundary propagators for the free vector, scalar, and spinor fields in the bulk at finite temperature. In Sec. IV we develop the idea of chiral condensate and the same time breaking of chiral symmetry at finite temperature. In Section VI we write the Lagrangian for the vector-spinor interaction in the bulk and using holographic correspondence obtain the temperature dependent integral expression for the  $g_{\omega NN}(T)$  at the boundary QCD. In Section VII we fix parameters and we plot graphics of  $g_{\omega NN}(T)$  and in section VIII we discuss results.

## 2. Soft-wall model at finite temperature

In general, in the soft-wall model of AdS/QCD at finite temperature the dilaton field  $\phi(z)$  can be considered as depending on temperature  $\phi(z, T)$  and the action for this model will be written in terms of such dilaton:

$$S = \int d^4 dz \sqrt{g} e^{-\phi(z, T)} L(x, z, T) \quad (1)$$

where  $g$  denotes  $g = |\det g_{MN}|$  ( $M, N = 0, 1, 2, 3, 5$ ), the extra dimension  $z$  varies in the range  $0 < z < \infty$  and an exponential factor is to make the integral over the  $z$  coordinate be finite at the IR boundary ( $z \rightarrow \infty$ ) when the parameter  $k$  is a scale parameter. AdS-Schwarzschild metric [2]:

$$ds^2 = e^{2A(z)} \left[ -f(z, T) dt^2 - (d\vec{x})^2 - \frac{dz^2}{f(z, T)} \right]$$

$$f(z, T) = 1 - \frac{z^4}{z_H^4} \quad (2)$$

where  $z_H$  is the position of the event horizon and it is related to the Hawking temperature as  $T = 1/(\pi z_H)$ ,  $x = (t, \vec{x})$  is the set of Minkowski coordinates,  $A(z) = \log(\frac{R}{z})$ , and  $R$  is the AdS space radius. In the approach for the finite temperature soft-wall model in Ref [25–27] the dilaton field  $\phi$  has been chosen temperature dependent through the temperature dependence of the dilaton parameter  $k^2$ :

$$\phi(z, T) = K^2(T) z^2, \quad (3)$$

when

$$K^2(T) = k^2 [1 + \Delta(T)]. \quad (4)$$

So,  $K^2(T)$  is the parameter of spontaneous breaking of chiral symmetry and the thermal function  $\Delta(T)$  up to  $T^4$  order has a form:

$$\Delta(T) = \delta_{T_1} \frac{T^2}{12F^2} + \delta_{T_2} \left( \frac{T^2}{12F^2} \right)^2 \quad (5)$$

Here  $F$  is the pseudoscalar decay constant in the chiral limit and the coefficients  $\delta_{T_1}$  and  $\delta_{T_2}$  are defined by the number of quark flavors  $N_f$

$$\delta_{T_1} = -\frac{N_f^2-1}{N_f}, \tag{6}$$

and

$$\delta_{T_1} = -\frac{N_f^2-1}{2N_f^2}. \tag{7}$$

### 3. The $\omega$ meson profile function at finite temperature

The vector field  $M_N(x,z,T)$ , which on the ultraviolet boundary of space-time gives the wave function of  $\omega$  meson, is composed from the gauge fields  $A_L$  and  $A_R$ :  $M_N = 1/2(A_L + A_R)$ . These gauge fields belong to the flavor symmetry subgroups  $SU(2)_L$  and  $SU(2)_R$ , which are part of the  $SU(2)_L \times SU(2)_R$  flavor group of the model. From these chiral gauge fields it is composed of an axial vector field as well. The action reads as below:.

$$S_M = -\frac{1}{2} \int_0^\infty d^4 x dz \sqrt{g} e^{-\varphi(z,T)} [\partial_N M_N(x, z, T) \partial^N M^N(x, z, T) - (\mu^2(z, T) + V(z, T)) M_N(x, z, T) M^N(x, z, T)] \tag{8}$$

Here  $V(z,T)$  is the thermal dilaton potential and it has expression below:

$$V(z, T) = \frac{e^{-2A(z)}}{f^{\frac{3}{5}}(z,T)} [\varphi''(z, T) + \varphi'(z, T)A'(z)] \tag{9}$$

where prime denotes  $z$  derivative. The temperature dependent bulk "mass"  $\mu(z,T)$  of the boson field  $M_N$  is

$$\mu^2(z, T) = \frac{\mu^2}{f^{\frac{3}{5}}(z,T)}. \tag{10}$$

The five dimensional mass  $\mu^2$  is expressed by means of the conformal dimension

$\Delta = N + L$  of the interpolating operator dual to the meson.  $N$  is the number of partons and  $L = \max|L_z|$  is the quark orbital angular momentum. For our  $\omega$  meson  $N = 2$  and  $L = 0$  for meson ground state. For this meson spin  $J = 1$  and the expression for  $\mu^2 R^2$  obtains simple form [25]:

$$\mu^2 R^2 = (\Delta - 1)(\Delta - 3). \tag{11}$$

The  $M_N$  and the Klauza - Klein (KK) expansion is performed:

$$M_\mu(x, z, T) = \sum_n M_{\mu n}(x) \Phi_n(z, T) \tag{12}$$

$M_{\mu n}(x)$  is KK modes wave functions corresponding to meson states,  $\Phi_n(z,T)$  are the temperature depending profile functions and  $n$  is the radial quantum number.

EOM for the  $\omega$  meson field will be reduced to the Schroedinger-type equation with the following replacement:

$\phi_n(z, T) = e^{\frac{-B(z,T)}{2}} \Phi_n(z, T)$  with  $B_T(z) = \phi(z, T) - A(z)$ . In the rest frame of the vector field the e.o.m. will give us following equation for  $\phi_n(z, T)$ :

$$\left[\frac{d^2}{dz^2} + U(z, T)\right] \phi_n(z, T) = M_n^2(T) \phi_n(z, T). \quad (13)$$

Here  $U(z, T)$  is the effective potential.

$$U(z, T) = U(z) + \Delta U(z, T). \quad (14)$$

Explicit forms of  $U(z)$  and  $\Delta U(z, T)$  are following form:

$$U(z) = k^4 z^2 + \frac{(4m^2 - 1)}{4z^2}, \quad (15)$$

$$\Delta U(z, T) = 2\Delta(T)k^4 z^2. \quad (16)$$

Here  $m = N + L - 2$  and for the  $\omega$  meson with two parton it equals to  $m = L$ . The meson mass spectrum  $M_n^2$  also is the sum of zero- and finite temperature parts at finite temperature:

$$M_n^2(T) = M_n^2(0) + \Delta M_n^2(T), \quad (17)$$

$$\Delta M_n^2(T) = \Delta(T)M_n^2(0) + \frac{R\pi^4 T^4}{k^2}, \quad (18)$$

$$M_n^2(0) = 4k^2\left(n + \frac{m+1}{2}\right), \quad R=(6n-1)(m+1). \quad (19)$$

At the end, the solution of equation (13) for the bulk profile  $\phi_n(z, T)$  was found in the following form [35]:

$$\phi_n(z, T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m+1)}} K^{m+1} z^{m+\frac{1}{2}} e^{-\frac{K^2 z^2}{2}} L_n^m(K^2 z^2). \quad (20)$$

### 3. Breaking of chiral symmetry at finite temperature

The pseudo-scalar field  $X$ , which transforms under the bifundamental representation of  $SU(2)_L \times SU(2)_R$  group, is introduced into the AdS/QCD models in order to perform breaking of the chiral  $SU(2)_L \times SU(2)_R$  symmetry group by Higgs mechanism in [29–31] and the action for this field has the form:

$$S_X = \int_0^\infty d^4x dz \sqrt{g} e^{-\varphi(z,T)} Tr[|DX|^2 + 3|X|^2] \quad (21)$$

Here  $D^M$  is the covariant derivative, which was defined as below:

$$D^M X = \partial^M X - iA_L^M X + XA_R^M = \partial^M X - i[M_M, X] - i\{A_M, X\}. \quad (22).$$

Since here we deal only with the vector field, we ignore the last term in the eq. (22). The solution of the equation of motion for the  $X$  field at zero temperature is widely described in the earlier works and we shall not repeat it here. We'll just recall here that the expected value for this field was found in the Ref. [29] as follows:

$$\langle X \rangle = \frac{1}{2} am_q z + \frac{1}{2a} \Sigma z^3 = v(z)$$

Here  $m_q$  is the mass of  $u$  and  $d$  quarks,  $\Sigma = \langle 0|q\bar{q}|0 \rangle$  is the value of chiral condensate at zero temperature and  $a = \sqrt{N_c} / (2\pi)$ .

$$\langle X(z, T) \rangle = \frac{1}{2} am_q z + \frac{1}{2a} \Sigma(T) z^3 = v(z, T). \quad (23)$$

In [25–27] it was supposed that the temperature dependence of the  $\Sigma(T) = \langle 0|q\bar{q}|0 \rangle_T$  quark condensate is identical to the temperature dependence of the dilaton parameter  $K^2(T)$ :

$$K^2(T) = k^2 \frac{\Sigma(T)}{\Sigma}. \quad (24)$$

Besides, it was conjectured that the relation at zero temperature between the quark condensate  $\Sigma$  and the number of flavors  $N_f$ , the condensate parameter  $B$  and the pseudo-scalar meson decay constant  $F$  in the chiral limit

$\Sigma = -N_f B F^2$  holds for the finite temperature case as well:

$$\Sigma(T) = -N_f B(T) F^2(T). \quad (25)$$

Then according to (4) and (24) we can write [27]:

$$\Sigma(T) = \Sigma [1 + \Delta(T)]. \quad (26)$$

Let us note that the relation (26) is valid until  $T^6$  degree of the temperature. The  $F(T)$  and  $B(T)$  dependencies have been studied in [25].

#### 4. Baryon profile function at finite temperature

In AdS space we have two bulk fermion fields ( $N_1, N_2$ ) in order to describe two independent chiral components of nucleons [30, 31] on the boundary. The action for the thermal fermion field  $N(x, z, T)$  is written as below [26]:

$$S = \int_0^\infty d^4 x dz e^{-\varphi(z, T)} \sqrt{g} \bar{N}(x, z, T) D_\mp(z, T) N(x, z, T), \quad (27)$$

where the  $D_{\pm}(z)$  covariant derivative has an explicit form below:

$$D_{\mp}(z, T) = \frac{i}{2} \Gamma^M \left[ \partial_M + \frac{1}{4} \omega_M^{ab} [\Gamma_a, \Gamma_b] \right] \pm [\mu_F(z, T) - U_F(z, T)] \quad (28)$$

Here  $\mu_F(z, T)$  is the five dimensional "mass" of the thermal fermion field  $N(x, z, T)$  in the soft-wall model of AdS/QCD at finite temperature:

$$\mu_F(z, T) = \mu_F f^{\frac{3}{10}}(z, T). \quad (29)$$

At zero temperature case it is determined by the following equation

$$m = N_B + L - \frac{3}{2} \quad (30)$$

where  $N_B = 3$  is the number of partons in the composite fermion and  $L$  is the orbital angular momentum ( $L = 0$  for the nucleons considered here). The temperature dependent potential  $U_F(z, T)$  for the fermions is related with the zero temperature one:

$$U_F(z, T) = \varphi(z, T) / f^{\frac{3}{10}}(z, T) \quad (31)$$

and the non - zero component of spin connection  $\omega_M^{ab}$  is given by

$$\omega_M^{ab} = (\delta_M^a \delta_z^b - \delta_M^b \delta_z^a) z f^{\frac{1}{5}}(z, T). \quad (32)$$

The commutators of the Dirac matrices in (28) is  $\sigma^{MN} = [\Gamma^M, \Gamma^N]$ . These matrices are related with ones in the reference frame by the  $\Gamma^M = e_a^M \Gamma^a$  relation, where  $e_a^M = z \text{diag}\{\frac{1}{f(z, T)}, 1, 1, 1, -f(z, T)\}$  are the inverse vielbeins and we pass to the reference frame  $\Gamma^a$  matrices by help of them  $\Gamma^a = (\gamma^\mu, -i\gamma^5)$ . Using the axial gauge  $N_5(x, z, T) = 0$  we decompose the AdS fermion field into the

left- and right-chirality components

$$N(x, z, T) = N^R(x, z, T) + N^L(x, z, T), \quad (33)$$

which are defined as usual ones  $N^R(x, z, T) = \frac{1-\gamma^5}{2} N$ ,  $N^L(x, z, T) = \frac{1+\gamma^5}{2} N$  with properties

$\gamma^5 N^L = -N^L$ ,  $\gamma^5 N^R = N^R$ . Kaluza-Klein expansion for the four dimensional transverse components of the AdS fields will be written in the terms of the sum of the profile functions  $\Phi_n^{L,R}(z, T)$ , which are temperature-dependent as well:

$$N^{L,R}(x, z, T) = \sum_n N_n^{L,R}(x) \Phi_n^{L,R}(z, T) \quad (34)$$

For the nucleons we consider the  $L = 0$  case and the total angular momentum will be  $J = \frac{1}{2}$ . For

this case it is useful to write the  $\Phi_n^{L,R}(z, T)$  profiles with the prefactors:

$$\Phi_n^{L,R}(z, T) = e^{\frac{-3A(z)}{2}} F_n^{L,R}(z, T) \quad (35).$$

After the substitution of these profile functions into the equations of motion in the rest frame of nucleon ( $p \sim 0$ ), we get following form of e.o.m. [26] for the  $F_n^{L,R}(z, T)$  profile functions:

$$[\partial_z^2 + U_{L,R}(z, T)] F_n^{L,R}(z, T) = M_n^2(T) F_n^{L,R}(z, T). \quad (36)$$

The temperature dependent spectrum  $M_n^2(T)$  is given below and it similar to one in zero temperature case:

$$M_n^2(T) = 4K^2(T) \left( n + m + \frac{1}{2} \right) = 4k^2(1 + \Delta(T))(n + m + \frac{1}{2}) \quad (37)$$

$U(z, T)$  in the (36) equation is the effective potential at finite temperature for the fermion field and it can be decomposed into a zero and finite temperature dependent terms as below:

$$U_{L,R}(z, T) = U_{L,R}(z) + \Delta U_{L,R}(z, T),$$

$$\Delta U_{L,R}(z, T) = 2\Delta(T)k^2 \left( k^2 z^2 + m \mp \frac{1}{2} \right). \quad (38)$$

Here

$$m = N + L - \frac{3}{2}. \quad (39)$$

Solutions to the equations (36) are the finite temperature profile functions for nucleons [35]:

$$F_n^{L,R}(z, T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m_L, m_R+1)}} K^{m_L, m_R+1} z^{m_L, m_R + \frac{1}{2}} e^{-\frac{K^2(T)z^2}{2}} L_n^{m_L, m_R}(K^2(T)z^2). \quad (40)$$

where  $m_L, m_R \pm \frac{1}{2}$ . The profile functions  $\Phi_n(z, T)$  and  $F_n(z, T)$  obey normalization conditions [35].

## 5. The $\omega$ meson coupling constant at finite temperature

To derive the  $\omega$  meson-nucleon thermal coupling constant in the AdS/CFT soft wall model we use the Ref. [8, 15, 30, 31]. We write down lagrangian for the interaction in the bulk, between the thermal bulk vector and nucleons fields. Then identifying the bulk partition function in the AdS-Schwarzschild background with one for the thermal QCD we shall obtain the expression for the thermal nucleon current interacting with the thermal  $\omega$  meson. In the AdS space interaction action in the bulk of AdS-Schwarzschild space-time will be written as below:

$$S = \int_0^\infty d^4x dz e^{-\varphi(z,T)} L_{int} \quad (41)$$

The generating functional  $Z_{AdS}$  of the bulk theory is  $Z_{QCD}$  of the QCD theory on the boundary

of this space-time:

$$Z_{AdS} = e^{iS_{int}} = Z_{QCD}. \quad (42)$$

To find the nucleon current, which interacts with the  $\omega$  meson in the boundary QCD theory and taking variation from the bulk functional  $Z_{AdS}$  over the boundary value of the bulk vector field  $M_\mu^a(q)$ :

$$\langle J_\mu^a \rangle = -i \frac{\delta Z_{AdS}}{\delta M_\mu^a(q)} \Big|_{M_\mu^a} = 0 \quad (43)$$

There are several kinds of interactions  $L_{int}$  consists of terms describing these interactions. The first term is a minimal gauge interaction term of the vector field with the current of bulk fermions:

$$L_{MNN}^0(T) = ik_1 e_A^M [\bar{N}_1 \Gamma^A(M)_M N_1 - \bar{N}_2 \Gamma^A(M)_M N_2] \quad (44)$$

Next terms are connected with the bulk spinor field's five dimensional "magnetic moments", which are described by  $\Gamma^{MN}$ . 4 dimensional components of this tensor correspond to the magnetic moments of fermions in the reference frame. First of such terms is a 5- dimensional generalization of a usual 4-dimensional magnetic interaction:

$$\begin{aligned} L_{MNN}^1(T) &= ik_1 e_A^M e_B^N [\bar{N}_1 \Gamma^{AB}(F_L)_{MN} N_1 - \bar{N}_2 \Gamma^{AB}(F_R)_{MN} N_2 + h.c.] = \\ &= ik_1 e_A^M e_B^N [\bar{N}_1 \Gamma^{AB} F_{MN} N_1 - \bar{N}_2 \Gamma^{AB} F_{MN} N_2 + h.c.] + axial\ vector\ term, \end{aligned} \quad (45)$$

where  $F_{MN} = \partial_M M_N - \partial_N M_M$  is the field strength tensor of the  $M_N$  vector field. Second of such "magnetic moment" terms was constructed in [31] and has a form:

$$\begin{aligned} L_{MNN}^2(T) &= \frac{i}{2} k_2 e_A^M e_B^N [\bar{N}_1 X \Gamma^{AB}(F_R)_{MN} N_2 - \bar{N}_2 X^+ \Gamma^{AB}(F_L)_{MN} N_1 - h.c.] = \\ &= \frac{i}{2} k_2 e_A^M e_B^N [\bar{N}_1 X \Gamma^{AB} F_{MN} N_2 + \bar{N}_2 X^+ \Gamma^{AB} F_{MN} N_1 + h.c.] + axial\ vector\ term. \end{aligned} \quad (46)$$

Besides the "magnetic moment" interaction it includes an interaction with the  $X$  field. As we mentioned before, the bulk scalar field  $X$  changes the chirality of the boundary nucleons and is expressed with the quark condensate  $\Sigma$  in the boundary theory. In the boundary QCD theory this term describes the nucleon-  $\omega$  meson-quark condensate coupling with the change of chirality of the nucleons. The  $k_1$  and  $k_2$  constants were determined at zero-temperature [31]. Therefore, the total "magnetic" type lagrangian is

$$L_{MNN}(T) = L_{MNN}^{(1)}(T) + L_{MNN}^{(2)}(T). \quad (47).$$



Having explicit expressions of thermal profile functions , we can calculate the terms of thermal action in the momentum space (43) from these terms. This variation gives us a contribution of each lagrangian term to the nucleon current at finite temperature:

$$\langle J_\mu(p', p, T) \rangle = g_{\omega NN}(T) \int dp' dp \bar{u}(p') \gamma_\mu u(p), \quad (48)$$

here  $g_{\omega NN}(T)$  is the integral over the holographic coordinate  $z$ . The contribution of each lagrangian term to the  $g_{\omega NN}(T)$  constant is following form:

$g_{\omega NN}^{(0)nm}(T)$   $L_{MNN}$  lagrangian is denoted by) and its integral expression is equal to following one:

$$g_{\omega NN}^{(0)nm}(T) = \int_0^\infty \frac{dz}{z^4} e^{-K^2(T)z^2} M_0(z, T) [F_{1L}^{(n)}(z, T)F_{1L}^{(m)}(z, T) + F_{2L}^{(n)}(z, T)F_{2L}^{(m)}(z, T)] \quad (49)$$

We have used relations between the profile functions of the bulk fermion fields as

$$F_{1L}^{(s)}(z, T) = F_{2R}^{(s)}(z, T),$$

$$F_{1R}^{(s)}(z, T) = -F_{2L}^{(s)}(z, T),$$

which are right for parity even states of nucleons (the ground and first excited N(1440) states, which we shall consider in numerical analysis, are parity even states).  $M_0(z, T)$  is the profile function of a vector meson. In (45) and in (46) the  $\Gamma^{MN}F_{MN}$  matrix is the sum of two kinds of terms, which are  $\Gamma^{5\nu}F_{5\nu}$  and  $\Gamma^{\mu\nu}F_{\mu\nu}$ . In the total lagrangian  $L^0_{MNN}(T)$  (47) the  $\Gamma^{5\nu}F_{5\nu}$  terms contribute to the  $g_{\omega NN}$  constant with the terms of derivative  $M_n^{(r)}(z, T)$  and the contribution of this term has following expression

at finite temperature:

$$g_{\omega NN}^{(1)nm}(T) = -2 \int_0^\infty \frac{dz}{z^3} e^{-K^2(T)z^2} M_n^{(r)}(z, T) [k_1 (F_{1L}^{(n)}(z, T)F_{1L}^{(m)}(z, T) - F_{2L}^{(n)}(z, T)F_{2L}^{(m)}(z, T)) + k_2 v(z, T) (F_{1L}^{(n)}(z, T)F_{2L}^{(m)}(z, T) - F_{2L}^{(n)}(z, T)F_{1L}^{(m)}(z, T))]. \quad (50)$$

The contribution of the  $\Gamma^{\mu\nu}F_{\mu\nu}$  terms is the following:

$$f_{\omega NN}^{nm}(T) = -4m_N \int_0^\infty \frac{dz}{z^3} e^{-K^2(T)z^2} M_0(z, T) [k_1 (F_{1L}^{(n)}(z, T)F_{1R}^{(m)}(z, T) - F_{2L}^{(n)}(z, T)F_{2R}^{(m)}(z, T)) + k_2 v(z, T) (F_{1L}^{(n)}(z, T)F_{2R}^{(m)}(z, T) - F_{2L}^{(n)}(z, T)F_{1R}^{(m)}(z, T))]. \quad (51)$$

Here  $m_N$  is the mass of the nucleon and  $f_{\omega NN}^{nm}(T)$  is the contribution of the interaction with the  $\omega$  meson due to

the nucleon's magnetic moment at finite temperature. Total coupling constant

$g_{\omega NN}^{(s.w.)nm}(T)$  is the sum of two coupling constants at finite temperature:

$$g_{\omega NN}^{(s.w.)nm}(T) = g_{\omega NN}^{(0)nm}(T) + g_{\omega NN}^{(1)nm}(T). \quad (52)$$

The  $g_{\omega NN}^{(0)nm}(T)$  coupling constant is interpreted as "strong charge" and the  $f_{\omega NN}^{nm}(T)$  constant as a constant of the interaction of the  $\omega$  meson with the nucleon by means of magnetic moment of last at finite temperature.

## 6. Numerical analysis

The  $g_{\omega NN}^{(s.w.)nm}(T)$  coupling constant consists in numerical calculation of the integrals for the constants  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  and in numerically drawing their temperature dependencies by means of MATHEMATICA package. We present our numerical results for the choice of parameters for two flavor  $N_f = 2$ ,  $F = 87 \text{ MeV}$ , three flavor  $N_f = 3$ ,  $F = 100 \text{ MeV}$ . These sets of parameters were taken from [25] We have free parameters  $k$ ,  $k_1$ ,  $k_2$ ,  $m_q$  and  $\Sigma$ . The  $k$  parameter was fixed at the value  $k = 391 \text{ MeV}$ . Note that, the  $\rho$  [35] and  $\omega$  vector mesons have the same quantum numbers. The only difference is in their masses ( $m_\rho = 770 \text{ MeV}$ ,  $m_\omega = 782 \text{ MeV}$ ). The difference between profile function of  $\rho$  and  $\omega$  vector mesons occur to order the parameter  $k$ . Such as,  $M^2 = 4k^2(n + \nu + 1)$ , when we consider ground state ( $n=0$ ), then we get value of  $k = 391 \text{ MeV}$ . The parameters  $k_1$  and  $k_2$  were fixed at the values  $k_1 = -0.78 \text{ GeV}^3$ ,  $k_2 = 0.5 \text{ GeV}^3$  in the [31]. Here we do not consider these constants. The  $\Sigma = (368)^3 \text{ MeV}^3$  value and the  $m_q = 0.00145 \text{ MeV}$  value of these parameters were found from the fitting of the  $\pi$  meson mass [34]. To have an idea of relative contributions of different terms of Lagrangian, we present results

for the temperature dependencies of the  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  coupling constants sepa-

rately. In the Figures below, the blue graph curve represents the  $g_{\omega NN}^{(0)nm}(T)$ , the orange curve shows the

$g_{\omega NN}^{(1)nm}(T)$ , green curve shows the  $g_{\omega NN}^{(s.w.)nm}(T)$  and red one shows the  $f_{\omega NN}^{nm}(T)$  coupling constants at finite temperature. We also have considered these dependencies for the first excited state  $N(1440)$  of the nucleons and plotting graphs for the different number of flavors.

## 7. Discussion

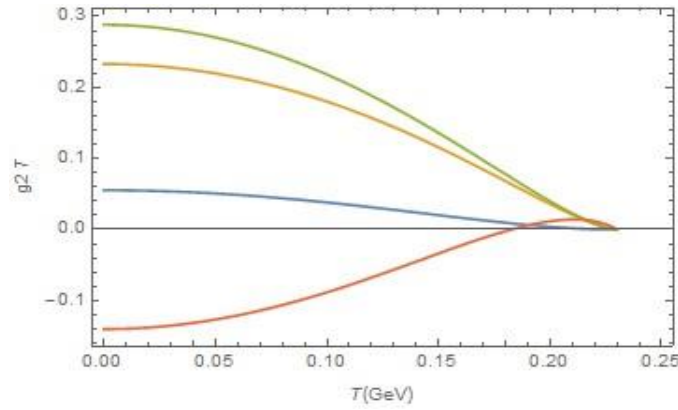
We studied strong coupling constant of the  $\omega$  meson with the nucleons within the soft-wall model of AdS/QCD at finite temperature. We have plotted this dependence for each term in the coupling constant and have observed that all terms become zero at the same point near the Hawking temperature. The result here is reasonable from a physical interpretation point of view and there are not any hadrons after this temperature, we get zero value for the coupling constant between hadrons behind this temperature. The graphics of  $g_{\omega NN}^{nm}(T)$  interpretation may be of use [12] for the understanding processes of early Universe.

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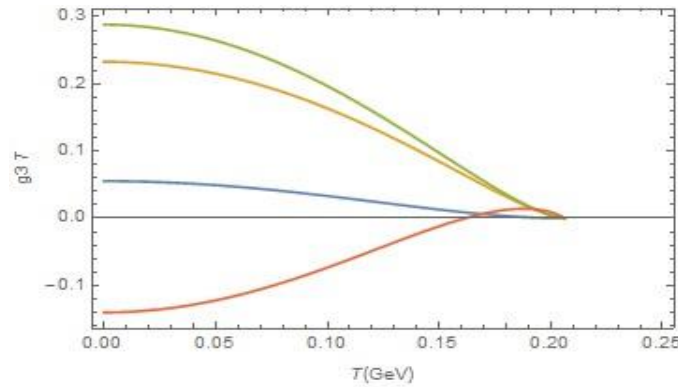
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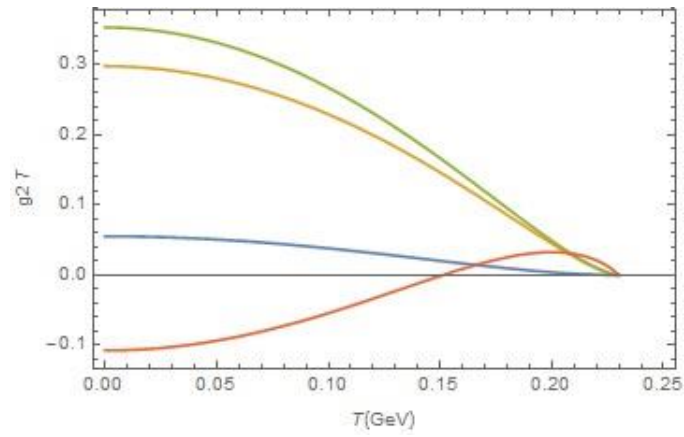
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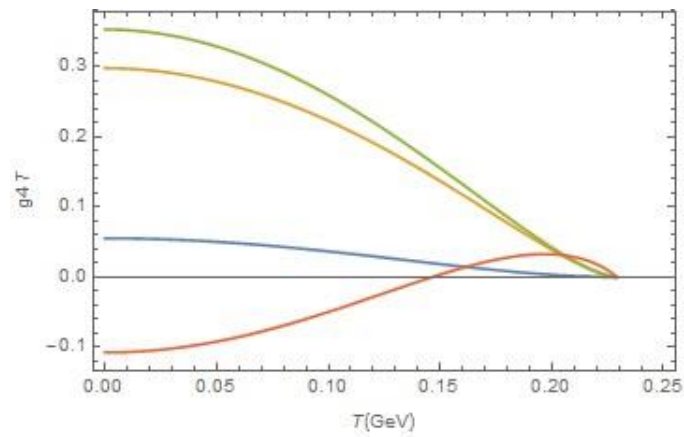
**Figure 1:** Comparison of  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$ ,  $g_{\omega NN}^{(s.w.)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  coupling constants at finite temperature for  $N_f = 2$ ,  $F = 87MEV$ .



**Figure 2:** Comparison of  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$ ,  $g_{\omega NN}^{(s.w.)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  coupling constants for  $N_f = 3$ ,  $F = 100MEV$ .



**Figure 3:** The  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$ ,  $g_{\omega NN}^{(s.w.)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  coupling constants for the first excited nucleons  $N(1440)$  at the parameter values  $N_f=2$ ,  $F=87\text{ MeV}$ .



**Figure 4:** The  $g_{\omega NN}^{(0)nm}(T)$ ,  $g_{\omega NN}^{(1)nm}(T)$ ,  $g_{\omega NN}^{(s.w.)nm}(T)$  and  $f_{\omega NN}^{nm}(T)$  coupling constants for the first excited nucleons  $N(1440)$  at the parameter values  $N_f=3$ ,  $F=100\text{ MeV}$ .